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AUTHOR Blackburn, Katie; White, David
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ABSTRACT

The concept of pi is one of great importance to all developed civilization and one that can be explored and mastered by elementary students through an inductive and problem-solving approach. Such an approach is outlined and discussed. The approach involves the following biblical quotation: "And he made a moltin sea ten cubits from one brim to the other: It was round all about, and its height was five cubits, and a line of thirty cubits did compass it round about." The quotation is reproduced on the board and students, in a problem-solving situation, attempt to try to find the value that the Hebrews attributed to pi. Materials needed for students to investigate this problem themselves inductively include straight pins, manilla folders, centimeter graph paper, envelopes, strip of colored tape, and large adhesive dots. In addition to detailed instructional strategies, extension and enrichment ideas are provided. These include measuring "around" and "across" circles with a flexible ruler and working with a mnemonic devised to remember pi to 30 decimal places. (JN)

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Circumference and Problem Solving

By Katie Blackburn and David White

How do we know that π, so commonly accepted an idea, is actually a given constant, an unvarying number believed by so many people around the globe? How can we be certain that this number (3.14) is the accurate figure?

In the two thousand years before the birth of Christ, the Babylonians expressed a value of pi; for them, its value was the number three. About three hundred years later, the Egyptians stated a value for this concept and gave the following fraction as the expression of pi: $(16/9)^2$. In China, too, the history of the believed value of pi reads as a listing of refinements and increasing accuracy:

1000 BC pi = 3
100 AD pi = 10
300 AD pi = 3 1/3
500 AD pi was computed to six decimal places

We can easily see, then, that the usefulness of having an accurate representation of pi was important to developing civilizations. The Hebrews went so far as to elevate the importance of the concept of pi to the status of being worthy to record in their religious writings. Reproduce the following Biblical quotation on the board and have students attempt a problem-solving situation by trying to find the value that the Hebrews attributed to pi.

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> And he made a molten sea ten cubits from one brim to the other: It was round all about, and its height was five cubits, and a line of thirty cubits did compass it round about.
>
> -1 Kings 7:23 (King James Version)

Lead the students through this problem solving situation by asking the following: In the above quote,

1) Ask them to find the measure of the circumference.
   (30 cubits)
2) Ask them to find the measure of the diameter.
   (10 cubits)
3) Ask them to find the measure of the radius.
   (5 cubits)
4) Ask them to find how many diameters it would take to measure out the circumference.
   (3 diameters)
5) Ask them to find how many radii it would take to measure out the circumference.
   (6 radii)

This brief introduction to the importance of the concept and the attributed value of pi should prepare youngsters to begin their own computation of the value of pi.

In order to help students investigate this problem themselves inductively, you will need to assemble the following materials: one straight pin per child, about 1/4 as many old manilla file folders as you have youngsters, one sheet of centimeter graph paper per child, one envelope per child, one long strip of colored tape and three large adhesive dots that would be visible on the

on the board from all points in the classroom.

First, on the board draw the grid for a graph so that it will be visible to all youngsters in the room. Label the horizontal axis as diameter and the vertical axis as circumference. Show the students three large circles that you have cut out of cardboard; holding up the circles, ask them how they might measure the distance across the circle. Most students, of course, will be able to contribute some offering of the use of a measuring stick; measure across the circles. Then ask them how they might measure the outside edge of the circle with only the available materials. This, of course, will call upon the ingenuity of the class and upon their problem solving abilities and resourcefulness. Lead the class through a thorough discussion of the drawbacks and advantages of any suggested possibilities (a straightedge won't bend unless it is especially made for flexibility; we don't have a string given to us that we could place along the outside edge; cuting the circle apart and trying to straighten out the resulting paper rim would not be very likely to work). One solution to the problem lies in the use of the graph on the board.

Hold up the smallest circle. Measure its diameter (let's say that it is six inches). Mark a starting point on the rim of the circle. Have a student roll the circle along the appropriate vertical line on the grid so that it makes one complete revolution and mark with one of the large adhesive dots the place where the starting point touches the line again (about 18). (The appropriate vertical line is found by locating the diameter

of the circle on the horizontal axis of the graph, in this case, six.) If an axle is desirable, use a pencil through the center of the circle. Discuss with the students whether or not the outside edge of the circle has been measured, that is, the circumference.

Now hold up a larger circle. Measure its diameter (let's say that its diameter is 10 inches). Placing the circle on the board (after drawing a starting line on the rim of the circle) at 10 on the horizontal axis, have the student roll out the circle's circumference (a little more than 30) and place a second large dot on the grid.

Repeat the experiment a third time with a larger circle, say one with a diameter of thirteen inches, and place the large dot on the board at the resulting measure (a little more than 39).

Figure 1

Now the students are ready to reproduce at their seats the same experiment. Have ready the following materials:

1) Sets of cardboard circles for each student. These can be easily prepared by filling the page of a ditto master with circles of varying sizes. Run the ditto master off onto the file folders by cutting the file folders at the fold and putting them into the usual paper feed on the ditto machine.

2) Cut out these circles and place them in an envelope.

3) Fold down the envelope and stick a straight pin through the flap so that the envelopes and circles can be used over and over.

4) Give one envelope with pin and circles to each student; also distribute centimeter graph paper.

The students are now ready to reproduce the board experiment. They should be instructed to mark a starting point on the rim of each of the circles. They should then be told to measure the diameter of each circle and label it. Using the pin as an axle, they should then roll the circles along the appropriate vertical line on their graph paper which corresponds to the measure of the diameter, plot the point, and repeat the process until at least three points have been plotted.

Bring the class attention back on the board by placing the strip of colored tape on the graph so that it makes a line through all the large adhesive dots. Have the students draw a similar straight line on their graph paper through the points they have plotted and extend the line.

Have the students roll other circular objects up the graph on the board such as records, plates, cans, rings, jar lids, and so on. The children should soon discover that it is not necessary to roll the circles up the graph. They should be able to find the circumference if they are given the diameter by finding the diameter on the horizontal axis and then determining the coordinate by moving up the vertical line until the graph of the line through the three points crosses

this vertical line. Likewise, the children should be able to determine the diameter if given the circumference by using coordinate graphing methods.

Students are now ready to manipulate the information about circles that they have generated. First, have students produce a bar graph that will restate the results of their line graph measurements. Then, have the students make a table or chart that lists their measuring results. The students will then be producing their own works that can be placed in acetate sidebinders and placed on classroom bookshelves. Their work would look something like the following:

Figure 2

No child can resist the excitement of being a published and read author.

From manipulating all this information about the measurement of circles, students will become familiar with the concept of the interrelation of circumference and diameter. Now, a few simple arithmetic steps are all that are needed for them to compute their values of pi. Lead the students in a discussion which results in the fact that there is a common factor in the results of all the measurements. That is, if they multiply the diameter by a little more than 3, they will obtain the circumference. The discussion might include the

following:

1) Ask students which is always larger, the diameter of a circle or its circumference.
2) Ask students about how many diameter lengths can be found in a circumference.
3) Ask students to divide the diameter into the circumference for each of the values they have found. Have them set this into a table that lists the diameter, the circumference, and the quotient of the division of the two.
4) Ask them to compare their quotients with the Hebrew, Chinese, and Egyptian values.
5) Ask them to compare among themselves their quotients.
6) List the students' values they found on the board. Find the value most often found by the class members.

Pi has now been discovered by the children to be 3.1, and is quite an accurate statement of its value. It remains only for the students to come up with a formula to derive the circumference of any circle: Write the following on the board and have the students supply the missing blanks:

The diameter times ____gives the circumference of a circle. (3.1 or 3.14)

d times _____ = the circumference (3.14)

Finally, they are ready to announce their mathematical finding in mathematical notation:

C = _____d (3.14) or $C = \pi d$

There are a number of extension and enrichment ideas which may be presented to some or all of the students, depending on their ability. The following might be considered:

1. Have the students measure "around" and "across" circles with a flexible ruler.
2. Discuss the fact that pi is irrational. Show a chart or poster with pi worked out to a large number of places. (Creative Publications has available a fine one where pi is computed to 8128 decimal places.)
3. Present the following mnemonic devised by A.C. Orr in 1906 for remembering pi to 30 decimal places. (Replace each word by the number of letters it contains.)

   Now I, even I, would celebrate
   In rhymes unapt, the great
   Immortal Syracusan, rivaled nevermore,
   Who in his wondrous lore,
   Passed on before,
   Left men his guidance
   How to circles mensurate.

   Another mnemonic, published in the Scientific American in 1914 is: "See, I have a rhyme assisting my feeble brain, its tasks ofttimes resisting."

4. Have students read about how the Egyptians, Greeks, Chinese, and Japanese calculated pi. One good source

Petr Beckmann's A History of π (St. Martin's Press, 1971).

5. Read about how the formulas for the area of a circle and the surface area of a sphere can be derived.
6. Present problems of the following type:
   1. Jane has a circular tablecloth which has a diameter of 3 feet. She wants to sew fringe around it. How much fringe should she buy?
   2. Find the perimeter of the following figure:

Figure 3

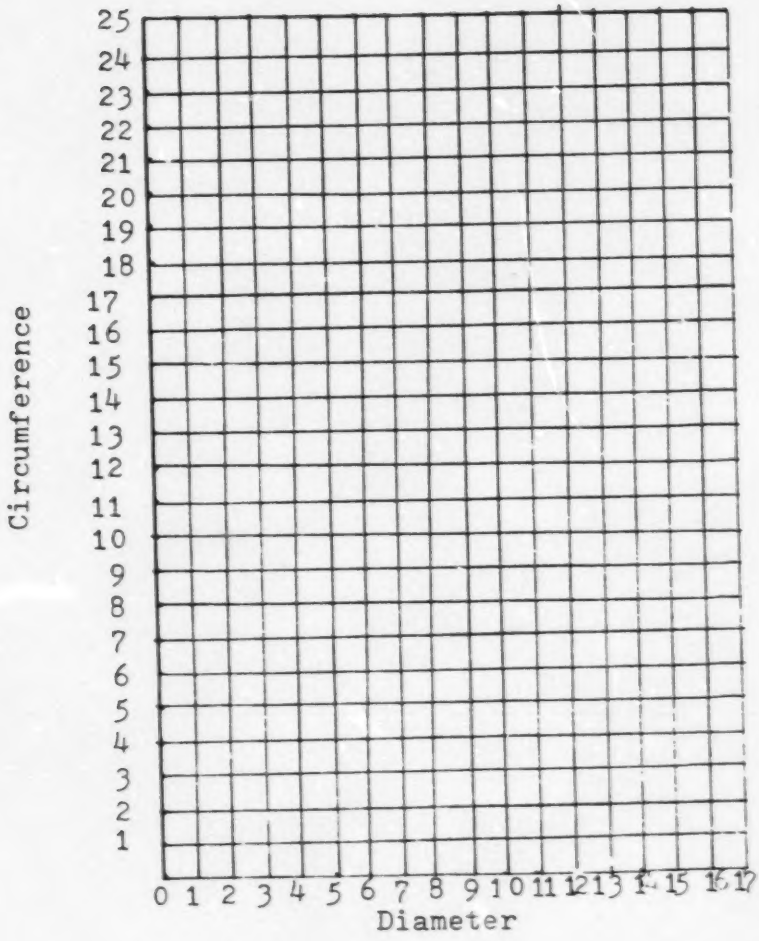
   3. If you wrapped kite string around a stick with a diameter of 4 inches, how could you figure out how much string you had if you wrapped the string around the stick with 4 turns?
   4. If Mrs. Jones made a giant pizza with a circumference of 10 feet, would it fit on the table which has a diameter of 4 feet?

Figure 4

The concept of pi is one of great importance to all developed civilizations and one that can be explored and mastered in the classroom by elementary students through an

inductive and problem solving approach. And, by the above procedure, not only have students discovered the value of pi for themselves, but they also have been introduced to the use of line graphs, bar graphs, tables, charts, and inductive proof. In addition, the more able students can delve into the topic with more depth and with more problem solving abilities. Few concepts as important as pi are as available or as fundamental, and using the above procedure, few are as rewarding for students in the elementary classroom.

Figure 1


Circumference
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16
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14
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12
11
10
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6
5
4
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2
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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
Diameter

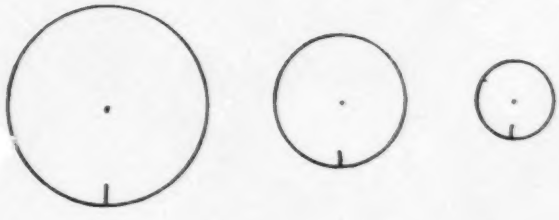
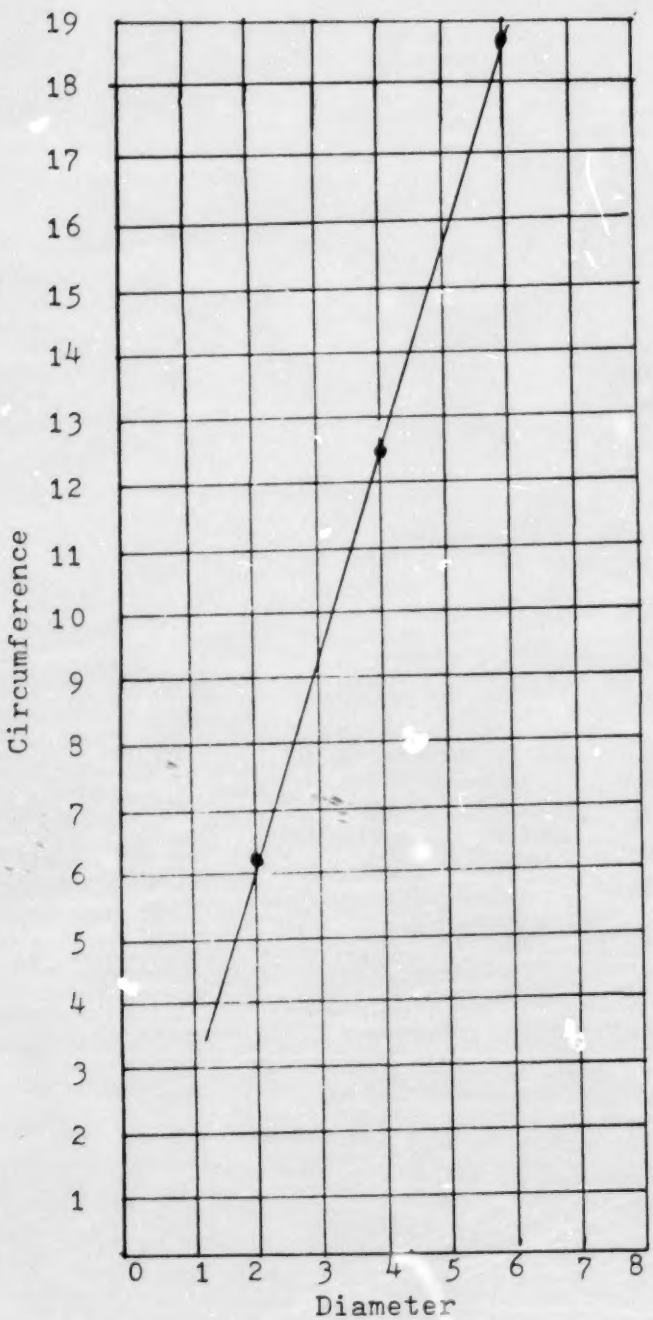


Figure 2



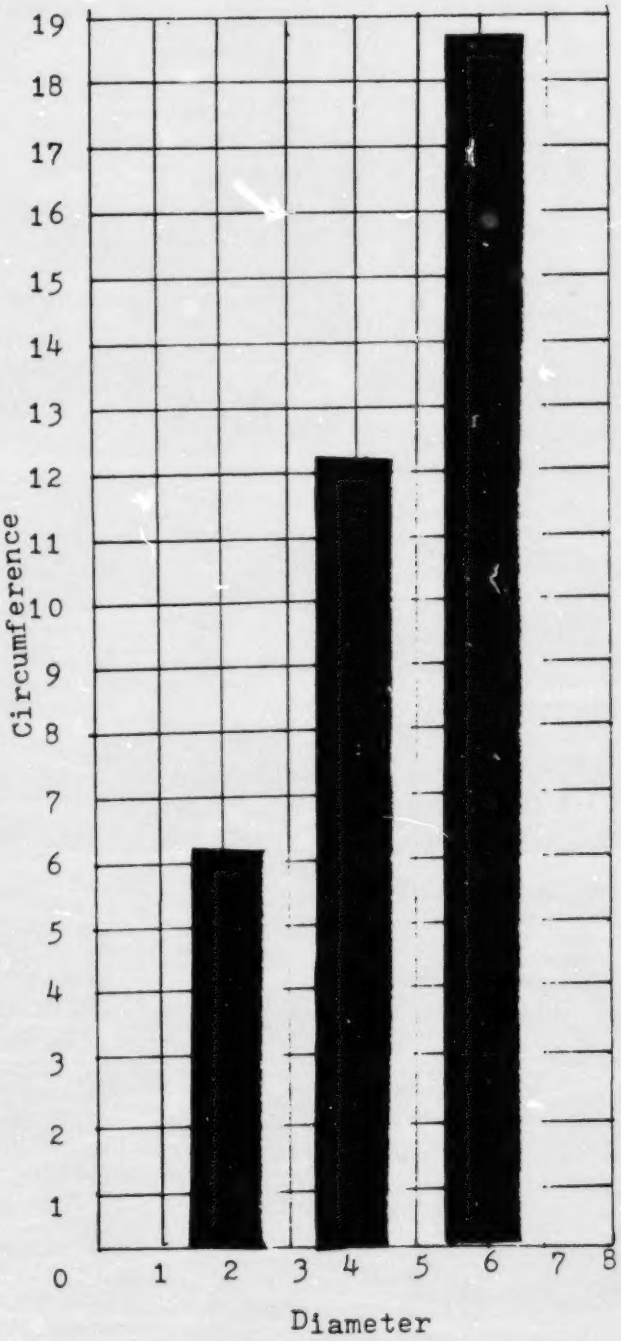


Table of Circles

Diameter	Circumference
2cm	about 6.3cm
4cm	about 12.6cm
6	about 18.8 cm

Figure 3

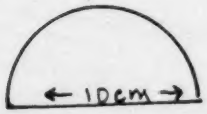


Figure 4

"Of course you smell pizza pie—
it's right there in front of you!"

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